



Syllabus

Cambridge International AS & A Level Mathematics 9709

For examination in June and November 2023, 2024 and 2025.
Also available for examination in March 2023, 2024 and 2025 for India only.



Why choose Cambridge?

Cambridge International prepares school students for life, helping them develop an informed curiosity and a lasting passion for learning. We are part of the University of Cambridge.

Our Cambridge Pathway gives students a clear path for educational success from age 5 to 19. Schools can shape the curriculum around how they want students to learn – with a wide range of subjects and flexible ways to offer them. It helps students discover new abilities and a wider world, and gives them the skills they need for life, so they can achieve at school, university and work.

Our programmes and qualifications set the global standard for international education. They are created by subject experts, rooted in academic rigour and reflect the latest educational research. They provide a strong platform for students to progress from one stage to the next, and are well supported by teaching and learning resources.

We review all our syllabuses regularly, so they reflect the latest research evidence and professional teaching practice – and take account of the different national contexts in which they are taught.

We consult with teachers to help us design each syllabus around the needs of their learners. Consulting with leading universities has helped us make sure our syllabuses encourage students to master the key concepts in the subject and develop the skills necessary for success in higher education.

Our mission is to provide educational benefit through provision of international programmes and qualifications for school education and to be the world leader in this field. Together with schools, we develop Cambridge learners who are confident, responsible, reflective, innovative and engaged – equipped for success in the modern world.

Every year, nearly a million Cambridge students from 10 000 schools in 160 countries prepare for their future with the Cambridge Pathway.

'We think the Cambridge curriculum is superb preparation for university.'

Christoph Guttentag, Dean of Undergraduate Admissions, Duke University, USA



Quality management

Cambridge International is committed to providing exceptional quality. In line with this commitment, our quality management system for the provision of international qualifications and education programmes for students aged 5 to 19 is independently certified as meeting the internationally recognised standard, ISO 9001:2015. Learn more at www.cambridgeinternational.org/ISO9001

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Important: Changes to this syllabus

For information about changes to this syllabus for 2023, 2024 and 2025, go to page 56.

The latest syllabus is version 1, published September 2020. There are no significant changes which affect teaching.

Any textbooks endorsed to support the syllabus for examination from 2020 are still suitable for use with this syllabus.

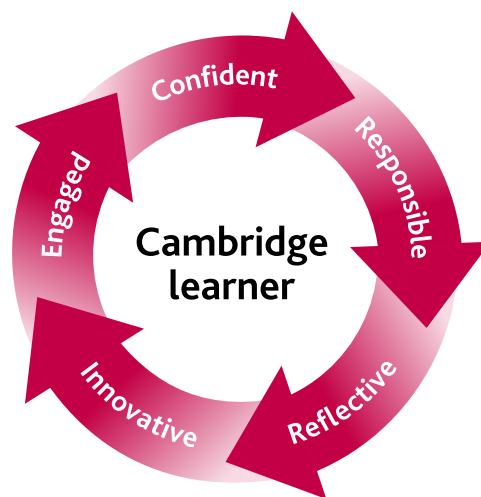
1 Why choose this syllabus?

Key benefits

The best motivation for a student is a real passion for the subject they're learning. By offering students a variety of Cambridge International AS & A Levels, you can give them the greatest chance of finding the path of education they most want to follow. With over 50 subjects to choose from, students can select the ones they love and that they're best at, which helps motivate them throughout their studies.

Following a Cambridge International AS & A Level programme helps students develop abilities which universities value highly, including:

- a deep understanding of their subjects
- higher order thinking skills – analysis, critical thinking, problem solving
- presenting ordered and coherent arguments
- independent learning and research.



Cambridge International AS & A Level Mathematics develops a set of transferable skills. These include the skill of working with mathematical information, as well as the ability to think logically and independently, consider accuracy, model situations mathematically, analyse results and reflect on findings. Learners can apply these skills across a wide range of subjects and the skills equip them well for progression to higher education or directly into employment.

Our approach in Cambridge International AS & A Level Mathematics encourages learners to be:

confident, using and sharing information and ideas, and using mathematical techniques to solve problems. These skills build confidence and support work in other subject areas as well as in mathematics.

responsible, through learning and applying skills which prepare them for future academic studies, helping them to become numerate members of society.

reflective, through making connections between different branches of mathematics and considering the outcomes of mathematical problems and modelling.

innovative, through solving both familiar and unfamiliar problems in different ways, selecting from a range of mathematical and problem-solving techniques.

engaged, by the beauty and structure of mathematics, its patterns and its many applications to real life situations.

'Cambridge students develop a deep understanding of subjects and independent thinking skills.'

Tony Hines, Principal, Rockledge High School, USA

Key concepts

Key concepts are essential ideas that help students develop a deep understanding of their subject and make links between different aspects. Key concepts may open up new ways of thinking about, understanding or interpreting the important things to be learned.

Good teaching and learning will incorporate and reinforce a subject's key concepts to help students gain:

- a greater depth as well as breadth of subject knowledge
- confidence, especially in applying knowledge and skills in new situations
- the vocabulary to discuss their subject conceptually and show how different aspects link together
- a level of mastery of their subject to help them enter higher education.

The key concepts identified below, carefully introduced and developed, will help to underpin the course you will teach. You may identify additional key concepts which will also enrich teaching and learning.

The key concepts for Cambridge International AS & A Level Mathematics are:

- **Problem solving**

Mathematics is fundamentally problem solving and representing systems and models in different ways. These include:

- Algebra: this is an essential tool which supports and expresses mathematical reasoning and provides a means to generalise across a number of contexts.
- Geometrical techniques: algebraic representations also describe a spatial relationship, which gives us a new way to understand a situation.
- Calculus: this is a fundamental element which describes change in dynamic situations and underlines the links between functions and graphs.
- Mechanical models: these explain and predict how particles and objects move or remain stable under the influence of forces.
- Statistical methods: these are used to quantify and model aspects of the world around us. Probability theory predicts how chance events might proceed, and whether assumptions about chance are justified by evidence.

- **Communication**

Mathematical proof and reasoning is expressed using algebra and notation so that others can follow each line of reasoning and confirm its completeness and accuracy. Mathematical notation is universal. Each solution is structured, but proof and problem solving also invite creative and original thinking.

- **Mathematical modelling**

Mathematical modelling can be applied to many different situations and problems, leading to predictions and solutions. A variety of mathematical content areas and techniques may be required to create the model. Once the model has been created and applied, the results can be interpreted to give predictions and information about the real world.

Recognition and progression

Our expertise in curriculum, teaching and learning, and assessment is the basis for the recognition of our programmes and qualifications around the world. Every year thousands of students with Cambridge International AS & A Levels gain places at leading universities worldwide. Our programmes and qualifications are valued by top universities around the world including those in the UK, US (including Ivy League universities), Europe, Australia, Canada and New Zealand.

UK NARIC, the national agency in the UK for the recognition and comparison of international qualifications and skills, has carried out an independent benchmarking study of Cambridge International AS & A Level and found it to be comparable to the standard of AS & A Level in the UK. This means students can be confident that their Cambridge International AS & A Level qualifications are accepted as equivalent, grade for grade, to UK AS & A Levels by leading universities worldwide.

Cambridge International AS Level Mathematics makes up the first half of the Cambridge International A Level course in mathematics and provides a foundation for the study of mathematics at Cambridge International A Level. Depending on local university entrance requirements, students may be able to use it to progress directly to university courses in mathematics or some other subjects. It is also suitable as part of a course of general education.

Cambridge International A Level Mathematics provides a foundation for the study of mathematics or related courses in higher education. Equally it is suitable as part of a course of general education.

For more information about the relationship between the Cambridge International AS Level and Cambridge International A Level see the 'Assessment overview' section of the Syllabus overview.

We recommend learners check the Cambridge recognitions database and the university websites to find the most up-to-date entry requirements for courses they wish to study.

Learn more at www.cambridgeinternational.org/recognition

Progression to Cambridge International AS & A Level Further Mathematics (9231)

Cambridge International AS & A Level Mathematics provides a good preparation for study of Cambridge International AS & A Level Further Mathematics (9231). Teachers should be aware that there are recommended combinations of components to study in Cambridge International AS & A Level Mathematics (9709) to support progression to Cambridge International AS & A Level Further Mathematics (9231).

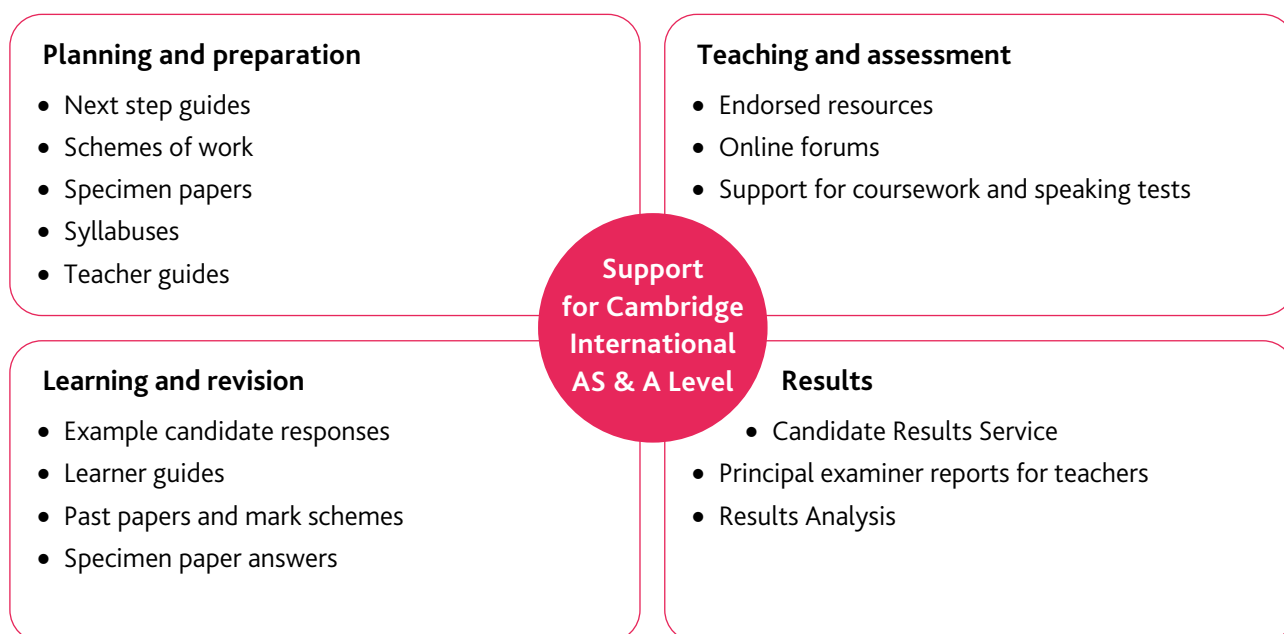
When planning a course which supports progression to Cambridge International AS & A Level Further Mathematics teachers should refer to the Cambridge International AS & A Level Further Mathematics syllabus and refer to the support document *Guide to prior learning for Paper 4 Further Probability and Statistics* on the Cambridge International website for more information.

Supporting teachers

We provide a wide range of resources, detailed guidance and innovative training and professional development so that you can give your students the best possible preparation for Cambridge International AS & A Level. To find out which resources are available for each syllabus go to our School Support Hub.

The School Support Hub is our secure online site for Cambridge teachers where you can find the resources you need to deliver our programmes. You can also keep up to date with your subject and the global Cambridge community through our online discussion forums.

Find out more at www.cambridgeinternational.org/support



Sign up for email notifications about changes to syllabuses, including new and revised products and services at www.cambridgeinternational.org/syllabusupdates

Professional development

We support teachers through:

- Introductory Training – face-to-face or online
- Extension Training – face-to-face or online
- Enrichment Professional Development – face-to-face or online

Find out more at www.cambridgeinternational.org/events

- Cambridge Professional Development Qualifications

Find out more at www.cambridgeinternational.org/profdev



Supporting exams officers

We provide comprehensive support and guidance for all Cambridge exams officers. Find out more at: www.cambridgeinternational.org/eoguide

2 Syllabus overview

Aims

The aims describe the purposes of a course based on this syllabus.

The aims are to enable students to:

- develop their mathematical knowledge and skills in a way which encourages confidence and provides satisfaction and enjoyment
- develop an understanding of mathematical principles and an appreciation of mathematics as a logical and coherent subject
- acquire a range of mathematical skills, particularly those which will enable them to use applications of mathematics in the context of everyday situations and of other subjects they may be studying
- develop the ability to analyse problems logically
- recognise when and how a situation may be represented mathematically, identify and interpret relevant factors and select an appropriate mathematical method to solve the problem
- use mathematics as a means of communication with emphasis on the use of clear expression
- acquire the mathematical background necessary for further study in mathematics or related subjects.

Cambridge Assessment International Education is an education organisation and politically neutral. The contents of this syllabus, examination papers and associated materials do not endorse any political view. We endeavour to treat all aspects of the exam process neutrally.



Content overview

Content section	Assessment component	Topics included
1 Pure Mathematics 1	Paper 1	1.1 Quadratics 1.2 Functions 1.3 Coordinate geometry 1.4 Circular measure 1.5 Trigonometry 1.6 Series 1.7 Differentiation 1.8 Integration
2 Pure Mathematics 2	Paper 2	2.1 Algebra 2.2 Logarithmic and exponential functions 2.3 Trigonometry 2.4 Differentiation 2.5 Integration 2.6 Numerical solution of equations
3 Pure Mathematics 3	Paper 3	3.1 Algebra 3.2 Logarithmic and exponential functions 3.3 Trigonometry 3.4 Differentiation 3.5 Integration 3.6 Numerical solution of equations 3.7 Vectors 3.8 Differential equations 3.9 Complex numbers
4 Mechanics	Paper 4	4.1 Forces and equilibrium 4.2 Kinematics of motion in a straight line 4.3 Momentum 4.4 Newton's laws of motion 4.5 Energy, work and power
5 Probability & Statistics 1	Paper 5	5.1 Representation of data 5.2 Permutations and combinations 5.3 Probability 5.4 Discrete random variables 5.5 The normal distribution
6 Probability & Statistics 2	Paper 6	6.1 The Poisson distribution 6.2 Linear combinations of random variables 6.3 Continuous random variables 6.4 Sampling and estimation 6.5 Hypothesis tests

Structure

There are six Mathematics components available:

Pure Mathematics components:

Paper 1: Pure Mathematics 1

Paper 2: Pure Mathematics 2

Paper 3: Pure Mathematics 3

Mechanics components:

Paper 4: Mechanics

Probability & Statistics components:

Paper 5: Probability & Statistics 1

Paper 6: Probability & Statistics 2

Candidates take two components for AS Level Mathematics.

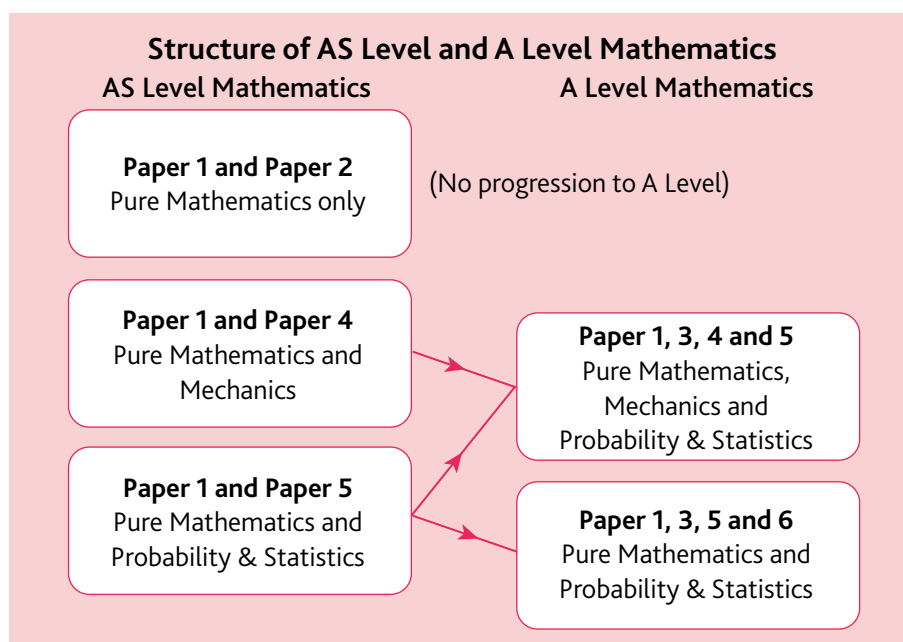
Candidates take four components for A Level Mathematics.

'The depth of knowledge displayed by the best A Level students makes them prime targets for America's Ivy League universities'

Yale University, USA

Three routes for Cambridge International AS & A Level Mathematics

Candidates following an AS Level route will be eligible for grades a–e. Candidates following an A Level route are eligible for grades A*–E.



This syllabus gives you the flexibility to design a course that will interest, challenge and engage your learners.

You are responsible for selecting a suitable combination of components to support your learners' further study.

Teachers should be aware that there are recommended combinations of components to study in Cambridge International AS & A Level Mathematics (9709) to support progression to Cambridge International AS & A Level Further Mathematics (9231).

When planning a course which supports progression to Cambridge International AS & A Level Further Mathematics teachers should refer to the Cambridge International AS & A Level Further Mathematics syllabus and refer to the support document *Guide to prior learning for Paper 4 Further Probability and Statistics* on the Cambridge International website for more information.

AS Mathematics (AS Level only)

Candidates take two components in the same series.

Paper 1	and	Paper 2
Pure Mathematics 1		Pure Mathematics 2

(Please note, this route cannot count towards A Level)

OR

Paper 1	and	Paper 4
Pure Mathematics 1		Mechanics

OR

Paper 1	and	Paper 5
Pure Mathematics 1		Probability & Statistics 1

Please note, the Pure Mathematics only route (Paper 1 and Paper 2) is available at AS Level only. Candidates who take the Pure Mathematics only route cannot then use their AS result and carry forward to complete the A Level.

A Level Mathematics

Candidates take four components for Cambridge International A Level Mathematics.

All candidates take:

Paper 1: Pure Mathematics 1

and

Paper 3: Pure Mathematics 3.

Then, candidates take **either**:

Paper 4: Mechanics

and

Paper 5: Probability & Statistics 1

OR

Paper 5: Probability & Statistics 1

and

Paper 6: Probability & Statistics 2.

Please note, it is not possible to combine Paper 4 and Paper 6. This is because Paper 6 depends on prior knowledge of the subject content for Paper 5.

A Level Mathematics (Staged route)

Candidates take AS Level components in the first year and carry forward their AS Level result. They then take two more components in another series to complete the A Level.

Year 1 (AS Level)	Paper 1 Pure Mathematics 1	and	Paper 4 Mechanics
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then

Year 2 (A Level)	Paper 3 Pure Mathematics 3	and	Paper 5 Probability & Statistics 1
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OR

Year 1 (AS Level)	Paper 1 Pure Mathematics 1	and	Paper 5 Probability & Statistics 1
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then

Year 2 (A Level)	Paper 3 Pure Mathematics 3	and	Paper 4 Mechanics
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OR

Year 1 (AS Level)	Paper 1 Pure Mathematics 1	and	Paper 5 Probability & Statistics 1
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then

Year 2 (A Level)	Paper 3 Pure Mathematics 3	and	Paper 6 Probability & Statistics 2
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A Level Mathematics (Linear route)

Candidates take the A Level components in the same series.

Paper 1 Pure Mathematics 1	Paper 4 Mechanics
Paper 3 Pure Mathematics 3	Paper 5 Probability & Statistics 1

OR

Paper 1 Pure Mathematics 1	Paper 5 Probability & Statistics 1
Paper 3 Pure Mathematics 3	Paper 6 Probability & Statistics 2

Assessment overview

Pure Mathematics components

Paper 1

Pure Mathematics 1 1 hour 50 minutes
75 marks
10 to 12 structured questions based on the
Pure Mathematics 1 subject content
Written examination
Externally assessed
60% of the AS Level
30% of the A Level
Compulsory for AS Level and A Level

Paper 2

Pure Mathematics 2 1 hour 15 minutes
50 marks
6 to 8 structured questions based on the
Pure Mathematics 2 subject content
Written examination
Externally assessed
40% of the AS Level
Offered only as part of AS Level

Paper 3

Pure Mathematics 3 1 hour 50 minutes
75 marks
9 to 11 structured questions based on the
Pure Mathematics 3 subject content
Written examination
Externally assessed
30% of the A Level only
Compulsory for A Level

Mechanics components

Paper 4

Mechanics 1 hour 15 minutes
50 marks
6 to 8 structured questions based on the
Mechanics subject content
Written examination
Externally assessed
40% of the AS Level
20% of the A Level
Offered as part of AS Level and A Level

Probability & Statistics components

Paper 5		Paper 6	
Probability & Statistics 1	1 hour 15 minutes	Probability & Statistics 2	1 hour 15 minutes
50 marks		50 marks	
6 to 8 structured questions based on the Probability & Statistics 1 subject content		6 to 8 structured questions based on the Probability & Statistics 2 subject content	
Written examination		Written examination	
Externally assessed		Externally assessed	
40% of the AS Level		20% of the A Level only	
20% of the A Level		Offered only as part of A Level	
Compulsory for A Level			

'Cambridge International AS & A Levels prepare students well for university because they've learnt to go into a subject in considerable depth. There's that ability to really understand the depth and richness and the detail of a subject. It's a wonderful preparation for what they are going to face at university.'

US Higher Education Advisory Council

Assessment objectives

The assessment objectives (AOs) are:

AO1 Knowledge and understanding

- Show understanding of relevant mathematical concepts, terminology and notation
- Recall accurately and use appropriate mathematical manipulative techniques

AO2 Application and communication

- Recognise the appropriate mathematical procedure for a given situation
- Apply appropriate combinations of mathematical skills and techniques in solving problems
- Present relevant mathematical work, and communicate corresponding conclusions, in a clear and logical way

Weighting for assessment objectives

The approximate weightings ($\pm 5\%$) allocated to each of the assessment objectives (AOs) are summarised below.

Assessment objectives as an approximate percentage of each component

Assessment objective	Weighting in components %					
	Paper 1	Paper 2	Paper 3	Paper 4	Paper 5	Paper 6
AO1 Knowledge and understanding	55	55	45	55	55	55
AO2 Application and communication	45	45	55	45	45	45

Assessment objectives as an approximate percentage of each qualification

Assessment objective	Weighting in AS Level %	Weighting in A Level %
AO1 Knowledge and understanding	55	52
AO2 Application and communication	45	48

3 Subject content

The mathematical content for each component is detailed below. You can teach the topics in any order you find appropriate.

Information about calculator use and information about the relationships between syllabus components can be found in 4 Details of the assessment.

Notes and examples are included to clarify the subject content. Please note that these are examples only and examination questions may differ from the examples given.

Prior knowledge

Knowledge of the content of the Cambridge IGCSE® Mathematics 0580 (Extended curriculum), or Cambridge International O Level (4024/4029), is assumed. Candidates should be familiar with scientific notation for compound units, e.g. 5 m s^{-1} for 5 metres per second.

In addition, candidates should:

- be able to carry out simple manipulation of surds (e.g. expressing $\sqrt{12}$ as $2\sqrt{3}$ and $\frac{6}{\sqrt{2}}$ as $3\sqrt{2}$),
- know the shapes of graphs of the form $y = kx^n$, where k is a constant and n is an integer (positive or negative) or $\pm\frac{1}{2}$.

1 Pure Mathematics 1 (for Paper 1)

1.1 Quadratics

Candidates should be able to:

- carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$ and use a completed square form
- find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant
- solve quadratic equations, and quadratic inequalities, in one unknown
- solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic
- recognise and solve equations in x which are quadratic in some function of x .

Notes and examples

e.g. to locate the vertex of the graph of $y = ax^2 + bx + c$ or to sketch the graph

e.g. to determine the number of real roots of the equation $ax^2 + bx + c = 0$. Knowledge of the term 'repeated root' is included.

By factorising, completing the square and using the formula.

e.g. $x + y + 1 = 0$ and $x^2 + y^2 = 25$,
 $2x + 3y = 7$ and $3x^2 = 4 + 4xy$.

e.g. $x^4 - 5x^2 + 4 = 0$, $6x + \sqrt{x} - 1 = 0$,
 $\tan^2 x = 1 + \tan x$.

1 Pure Mathematics 1

1.2 Functions

Candidates should be able to:

- understand the terms function, domain, range, one-one function, inverse function and composition of functions
- identify the range of a given function in simple cases, and find the composition of two given functions
- determine whether or not a given function is one-one, and find the inverse of a one-one function in simple cases
- illustrate in graphical terms the relation between a one-one function and its inverse
- understand and use the transformations of the graph of $y = f(x)$ given by
 $y = f(x) + a$, $y = f(x + a)$,
 $y = af(x)$, $y = f(ax)$ and simple combinations of these.

Notes and examples

e.g. range of $f : x \mapsto \frac{1}{x}$ for $x \geq 1$ and

range of $g : x \mapsto x^2 + 1$ for $x \in \mathbb{R}$. Including the condition that a composite function gf can only be formed when the range of f is within the domain of g .

e.g. finding the inverse of

$$h : x \mapsto (2x + 3)^2 - 4 \text{ for } x < -\frac{3}{2}.$$

Sketches should include an indication of the mirror line $y = x$.

Including use of the terms 'translation', 'reflection' and 'stretch' in describing transformations. Questions may involve algebraic or trigonometric functions, or other graphs with given features.

1.3 Coordinate geometry

Candidates should be able to:

- find the equation of a straight line given sufficient information
- interpret and use any of the forms $y = mx + c$, $y - y_1 = m(x - x_1)$, $ax + by + c = 0$ in solving problems
- understand that the equation $(x - a)^2 + (y - b)^2 = r^2$ represents the circle with centre (a, b) and radius r
- use algebraic methods to solve problems involving lines and circles
- understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations.

Notes and examples

e.g. given two points, or one point and the gradient.

Including calculations of distances, gradients, midpoints, points of intersection and use of the relationship between the gradients of parallel and perpendicular lines.

Including use of the expanded form $x^2 + y^2 + 2gx + 2fy + c = 0$.

Including use of elementary geometrical properties of circles, e.g. tangent perpendicular to radius, angle in a semicircle, symmetry.

Implicit differentiation is not included.

e.g. to determine the set of values of k for which the line $y = x + k$ intersects, touches or does not meet a quadratic curve.

1 Pure Mathematics 1

1.4 Circular measure

Candidates should be able to:

- understand the definition of a radian, and use the relationship between radians and degrees
- use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ in solving problems concerning the arc length and sector area of a circle.

Notes and examples

Including calculation of lengths and angles in triangles and areas of triangles.

1.5 Trigonometry

Candidates should be able to:

- sketch and use graphs of the sine, cosine and tangent functions (for angles of any size, and using either degrees or radians)
- use the exact values of the sine, cosine and tangent of 30° , 45° , 60° , and related angles
- use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometric relations
- use the identities $\frac{\sin \theta}{\cos \theta} \equiv \tan \theta$ and $\sin^2 \theta + \cos^2 \theta \equiv 1$
- find all the solutions of simple trigonometrical equations lying in a specified interval (general forms of solution are not included).

Notes and examples

Including e.g. $y = 3 \sin x$, $y = 1 - \cos 2x$,
 $y = \tan\left(x + \frac{1}{4}\pi\right)$.

e.g. $\cos 150^\circ = -\frac{1}{2}\sqrt{3}$, $\sin \frac{3}{4}\pi = \frac{1}{\sqrt{2}}$.

No specialised knowledge of these functions is required, but understanding of them as examples of inverse functions is expected.

e.g. in proving identities, simplifying expressions and solving equations.

e.g. solve $3 \sin 2x + 1 = 0$ for $-\pi < x < \pi$,
 $3 \sin^2 \theta - 5 \cos \theta - 1 = 0$ for $0^\circ \leq \theta \leq 360^\circ$.

1.6 Series

Candidates should be able to:

- use the expansion of $(a + b)^n$, where n is a positive integer
- recognise arithmetic and geometric progressions
- use the formulae for the n th term and for the sum of the first n terms to solve problems involving arithmetic or geometric progressions
- use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression.

Notes and examples

Including the notations $\binom{n}{r}$ and $n!$

Knowledge of the greatest term and properties of the coefficients are not required.

Including knowledge that numbers a, b, c are 'in arithmetic progression' if $2b = a + c$ (or equivalent) and are 'in geometric progression' if $b^2 = ac$ (or equivalent).

Questions may involve more than one progression.

1 Pure Mathematics 1

1.7 Differentiation

Candidates should be able to:

- understand the gradient of a curve at a point as the limit of the gradients of a suitable sequence of chords, and use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$ for first and second derivatives
- use the derivative of x^n (for any rational n), together with constant multiples, sums and differences of functions, and of composite functions using the chain rule
- apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change
- locate stationary points and determine their nature, and use information about stationary points in sketching graphs.

Notes and examples

Only an informal understanding of the idea of a limit is expected.

e.g. includes consideration of the gradient of the chord joining the points with x coordinates 2 and $(2 + h)$ on the curve $y = x^3$. Formal use of the general method of differentiation from first principles is not required.

e.g. find $\frac{dy}{dx}$, given $y = \sqrt{2x^3 + 5}$.

Including connected rates of change, e.g. given the rate of increase of the radius of a circle, find the rate of increase of the area for a specific value of one of the variables.

Including use of the second derivative for identifying maxima and minima; alternatives may be used in questions where no method is specified.

Knowledge of points of inflexion is not included.

1.8 Integration

Candidates should be able to:

- understand integration as the reverse process of differentiation, and integrate $(ax + b)^n$ (for any rational n except -1), together with constant multiples, sums and differences
- solve problems involving the evaluation of a constant of integration
- evaluate definite integrals
- use definite integration to find
 - the area of a region bounded by a curve and lines parallel to the axes, or between a curve and a line or between two curves
 - a volume of revolution about one of the axes.

Notes and examples

e.g. $\int (2x^3 - 5x + 1)dx$, $\int \frac{1}{(2x + 3)^2} dx$.

e.g. to find the equation of the curve through $(1, -2)$ for which $\frac{dy}{dx} = \sqrt{2x + 1}$.

Including simple cases of 'improper' integrals, such as $\int_0^1 x^{-\frac{1}{2}} dx$ and $\int_1^{\infty} x^{-2} dx$.

A volume of revolution may involve a region not bounded by the axis of rotation, e.g. the region between $y = 9 - x^2$ and $y = 5$ rotated about the x -axis.

2 Pure Mathematics 2 (for Paper 2)

Knowledge of the content for Paper 1: Pure Mathematics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

2.1 Algebra

Candidates should be able to:

- understand the meaning of $|x|$, sketch the graph of $y = |ax + b|$ and use relations such as $|a| = |b| \Leftrightarrow a^2 = b^2$ and $|x - a| < b \Leftrightarrow a - b < x < a + b$ when solving equations and inequalities
- divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)
- use the factor theorem and the remainder theorem.

Notes and examples

Graphs of $y = |f(x)|$ and $y = f(|x|)$ for non-linear functions f are not included.

e.g. $|3x - 2| = |2x + 7|$, $2x + 5 < |x + 1|$

e.g. to find factors and remainders, solve polynomial equations or evaluate unknown coefficients.

Including factors of the form $(ax + b)$ in which the coefficient of x is not unity, and including calculation of remainders.

2.2 Logarithmic and exponential functions

Candidates should be able to:

- understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)
- understand the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs
- use logarithms to solve equations and inequalities in which the unknown appears in indices
- use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.

Notes and examples

Including knowledge of the graph of $y = e^{kx}$ for both positive and negative values of k .

e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4^{2x-1}$.

e.g.

$y = kx^n$ gives $\ln y = \ln k + n \ln x$ which is linear in $\ln x$ and $\ln y$

$y = k(a^x)$ gives $\ln y = \ln k + x \ln a$ which is linear in x and $\ln y$.

2 Pure Mathematics 2

2.3 Trigonometry

Candidates should be able to:

- understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude
- use trigonometrical identities for the simplification and exact evaluation of expressions, and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of
 - $\sec^2\theta \equiv 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$
 - the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$
 - the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$
 - the expression of $a \sin \theta + b \cos \theta$ in the forms $R \sin(\theta \pm \alpha)$ and $R \cos(\theta \pm \alpha)$.

Notes and examples

e.g. simplifying $\cos(x - 30^\circ) - 3 \sin(x - 60^\circ)$.
 e.g. solving $\tan \theta + \cot \theta = 4$, $2 \sec^2 \theta - \tan \theta = 5$,
 $3 \cos \theta + 2 \sin \theta = 1$.

2.4 Differentiation

Candidates should be able to:

- use the derivatives of e^x , $\ln x$, $\sin x$, $\cos x$, $\tan x$, together with constant multiples, sums, differences and composites
- differentiate products and quotients
- find and use the first derivative of a function which is defined parametrically or implicitly.

Notes and examples

e.g. $\frac{2x-4}{3x+2}$, $x^2 \ln x$, xe^{1-x^2} .

e.g. $x = t - e^{2t}$, $y = t + e^{2t}$.
 e.g. $x^2 + y^2 = xy + 7$.

Including use in problems involving tangents and normals.

2.5 Integration

Candidates should be able to:

- extend the idea of 'reverse differentiation' to include the integration of e^{ax+b} , $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$ and $\sec^2(ax+b)$
- use trigonometrical relationships in carrying out integration
- understand and use the trapezium rule to estimate the value of a definite integral.

Notes and examples

Knowledge of the general method of integration by substitution is not required.

e.g. use of double-angle formulae to integrate $\sin^2 x$ or $\cos^2(2x)$.

Including use of sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or an under-estimate.

2 Pure Mathematics 2

2.6 Numerical solution of equations

Candidates should be able to:

- locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change
- understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation
- understand how a given simple iterative formula of the form $x_{n+1} = F(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy.

Notes and examples

e.g. finding a pair of consecutive integers between which a root lies.

Knowledge of the condition for convergence is not included, but an understanding that an iteration may fail to converge is expected.

3 Pure Mathematics 3 (for Paper 3)

Knowledge of the content of Paper 1: Pure Mathematics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

3.1 Algebra

Candidates should be able to:

- understand the meaning of $|x|$, sketch the graph of $y = |ax + b|$ and use relations such as $|a| = |b| \Leftrightarrow a^2 = b^2$ and $|x - a| < b \Leftrightarrow a - b < x < a + b$ when solving equations and inequalities
- divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)
- use the factor theorem and the remainder theorem
- recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than
 - $(ax + b)(cx + d)(ex + f)$
 - $(ax + b)(cx + d)^2$
 - $(ax + b)(cx^2 + d)$
- use the expansion of $(1 + x)^n$, where n is a rational number and $|x| < 1$.

Notes and examples

Graphs of $y = |f(x)|$ and $y = f(|x|)$ for non-linear functions f are not included.

e.g. $|3x - 2| = |2x + 7|$, $2x + 5 < |x + 1|$.

e.g. to find factors and remainders, solve polynomial equations or evaluate unknown coefficients.

Including factors of the form $(ax + b)$ in which the coefficient of x is not unity, and including calculation of remainders.

Excluding cases where the degree of the numerator exceeds that of the denominator

Finding the general term in an expansion is not included.

Adapting the standard series to expand e.g. $(2 - \frac{1}{2}x)^{-1}$ is included, and determining the set of values of x for which the expansion is valid in such cases is also included.

3 Pure Mathematics 3

3.2 Logarithmic and exponential functions

Candidates should be able to:

- understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)
- understand the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs
- use logarithms to solve equations and inequalities in which the unknown appears in indices
- use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.

Notes and examples

Including knowledge of the graph of $y = e^{kx}$ for both positive and negative values of k .

e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4^{2x-1}$.

e.g.

$y = kx^n$ gives $\ln y = \ln k + n \ln x$ which is linear in $\ln x$ and $\ln y$.

$y = k(a^x)$ gives $\ln y = \ln k + x \ln a$ which is linear in x and $\ln y$.

3.3 Trigonometry

Candidates should be able to:

- understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude
- use trigonometrical identities for the simplification and exact evaluation of expressions, and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of
 - $\sec^2 \theta \equiv 1 + \tan^2 \theta$ and $\operatorname{cosec}^2 \theta \equiv 1 + \cot^2 \theta$
 - the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$
 - the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$
 - the expression of $a \sin \theta + b \cos \theta$ in the forms $R \sin(\theta \pm \alpha)$ and $R \cos(\theta \pm \alpha)$.

Notes and examples

e.g. simplifying $\cos(x - 30^\circ) - 3 \sin(x - 60^\circ)$.

e.g. solving $\tan \theta + \cot \theta = 4$, $2 \sec^2 \theta - \tan \theta = 5$, $3 \cos \theta + 2 \sin \theta = 1$.

3 Pure Mathematics 3

3.4 Differentiation

Candidates should be able to:

- use the derivatives of e^x , $\ln x$, $\sin x$, $\cos x$, $\tan x$, $\tan^{-1} x$, together with constant multiples, sums, differences and composites
- differentiate products and quotients
- find and use the first derivative of a function which is defined parametrically or implicitly.

Notes and examples

Derivatives of $\sin^{-1} x$ and $\cos^{-1} x$ are not required.

e.g. $\frac{2x-4}{3x+2}$, $x^2 \ln x$, xe^{1-x^2} .

e.g. $x = t - e^{2t}$, $y = t + e^{2t}$.

e.g. $x^2 + y^2 = xy + 7$.

Including use in problems involving tangents and normals.

3.5 Integration

Candidates should be able to:

- extend the idea of 'reverse differentiation' to include the integration of e^{ax+b} , $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$, $\sec^2(ax+b)$ and $\frac{1}{x^2+a^2}$
- use trigonometrical relationships in carrying out integration
- integrate rational functions by means of decomposition into partial fractions
- recognise an integrand of the form $\frac{kf'(x)}{f(x)}$, and integrate such functions
- recognise when an integrand can usefully be regarded as a product, and use integration by parts
- use a given substitution to simplify and evaluate either a definite or an indefinite integral.

Notes and examples

Including examples such as $\frac{1}{2+3x^2}$.

e.g. use of double-angle formulae to integrate $\sin^2 x$ or $\cos^2(2x)$.

Restricted to types of partial fractions as specified in topic 3.1 above.

e.g. integration of $\frac{x}{x^2+1}$, $\tan x$.

e.g. integration of $x \sin 2x$, $x^2 e^{-x}$, $\ln x$, $x \tan^{-1} x$.

e.g. to integrate $\sin^2 2x \cos x$ using the substitution $u = \sin x$.

3 Pure Mathematics 3

3.6 Numerical solution of equations

Candidates should be able to:

- locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change
- understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation
- understand how a given simple iterative formula of the form $x_{n+1} = F(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy.

Notes and examples

e.g. finding a pair of consecutive integers between which a root lies.

Knowledge of the condition for convergence is not included, but an understanding that an iteration may fail to converge is expected.

3.7 Vectors

Candidates should be able to:

- use standard notations for vectors, i.e. $\begin{pmatrix} x \\ y \end{pmatrix}, x\mathbf{i} + y\mathbf{j}, \begin{pmatrix} x \\ y \\ z \end{pmatrix}, x\mathbf{i} + y\mathbf{j} + z\mathbf{k}, \overrightarrow{AB}, \mathbf{a}$
- carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms
- calculate the magnitude of a vector, and use unit vectors, displacement vectors and position vectors
- understand the significance of all the symbols used when the equation of a straight line is expressed in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, and find the equation of a line, given sufficient information
- determine whether two lines are parallel, intersect or are skew, and find the point of intersection of two lines when it exists
- use formulae to calculate the scalar product of two vectors, and use scalar products in problems involving lines and points.

Notes and examples

e.g. ' $OABC$ is a parallelogram' is equivalent to $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{OC}$.

The general form of the ratio theorem is not included, but understanding that the midpoint of AB has position vector $\frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$ is expected.

In 2 or 3 dimensions.

e.g. finding the equation of a line given the position vector of a point on the line and a direction vector, or the position vectors of two points on the line.

Calculation of the shortest distance between two skew lines is not required. Finding the equation of the common perpendicular to two skew lines is also not required.

e.g. finding the angle between two lines, and finding the foot of the perpendicular from a point to a line; questions may involve 3D objects such as cuboids, tetrahedra (pyramids), etc.

Knowledge of the vector product is not required.

3 Pure Mathematics 3

3.8 Differential equations

Candidates should be able to:

- formulate a simple statement involving a rate of change as a differential equation
- find by integration a general form of solution for a first order differential equation in which the variables are separable
- use an initial condition to find a particular solution
- interpret the solution of a differential equation in the context of a problem being modelled by the equation.

Notes and examples

The introduction and evaluation of a constant of proportionality, where necessary, is included.

Including any of the integration techniques from topic 3.5 above.

Where a differential equation is used to model a 'real-life' situation, no specialised knowledge of the context will be required.

3.9 Complex numbers

Candidates should be able to:

- understand the idea of a complex number, recall the meaning of the terms real part, imaginary part, modulus, argument, conjugate, and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal
- carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in Cartesian form $x + iy$
- use the result that, for a polynomial equation with real coefficients, any non-real roots occur in conjugate pairs
- represent complex numbers geometrically by means of an Argand diagram
- carry out operations of multiplication and division of two complex numbers expressed in polar form $r(\cos \theta + i \sin \theta) \equiv re^{i\theta}$
- find the two square roots of a complex number
- understand in simple terms the geometrical effects of conjugating a complex number and of adding, subtracting, multiplying and dividing two complex numbers
- illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram

Notes and examples

Notations $\operatorname{Re} z$, $\operatorname{Im} z$, $|z|$, $\arg z$, z^* should be known. The argument of a complex number will usually refer to an angle θ such that $-\pi < \theta \leq \pi$, but in some cases the interval $0 \leq \theta < 2\pi$ may be more convenient. Answers may use either interval unless the question specifies otherwise.

For calculations involving multiplication or division, full details of the working should be shown.

e.g. in solving a cubic or quartic equation where one complex root is given.

Including the results $|z_1 z_2| = |z_1| |z_2|$ and $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$, and corresponding results for division.

e.g. the square roots of $5 + 12i$ in exact Cartesian form. Full details of the working should be shown.

e.g. $|z - a| < k$, $|z - a| = |z - b|$, $\arg(z - a) = \alpha$.

4 Mechanics (for Paper 4)

Questions set will be mainly numerical, and will aim to test mechanical principles without involving difficult algebra or trigonometry. However, candidates should be familiar in particular with the following trigonometrical results:

$$\sin(90^\circ - \theta) \equiv \cos \theta, \cos(90^\circ - \theta) \equiv \sin \theta, \tan \theta \equiv \frac{\sin \theta}{\cos \theta}, \sin^2 \theta + \cos^2 \theta \equiv 1.$$

Knowledge of algebraic methods from the content for Paper 1: Pure Mathematics 1 is assumed.

This content list refers to the equilibrium or motion of a 'particle'. Examination questions may involve extended bodies in a 'realistic' context, but these extended bodies should be treated as particles, so any force acting on them is modelled as acting at a single point.

Vector notation will not be used in the question papers.

4.1 Forces and equilibrium

Candidates should be able to:

- identify the forces acting in a given situation
- understand the vector nature of force, and find and use components and resultants
- use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero
- understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component
- use the model of a 'smooth' contact, and understand the limitations of this model
- understand the concepts of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F = \mu R$ or $F \leq \mu R$, as appropriate
- use Newton's third law.

Notes and examples

e.g. by drawing a force diagram.

Calculations are always required, not approximate solutions by scale drawing.

Solutions by resolving are usually expected, but equivalent methods (e.g. triangle of forces, Lami's Theorem, where suitable) are also acceptable; these other methods are not required knowledge, and will not be referred to in questions.

Terminology such as 'about to slip' may be used to mean 'in limiting equilibrium' in questions.

e.g. the force exerted by a particle on the ground is equal and opposite to the force exerted by the ground on the particle.

4 Mechanics

4.2 Kinematics of motion in a straight line

Candidates should be able to:

- understand the concepts of distance and speed as scalar quantities, and of displacement, velocity and acceleration as vector quantities
- sketch and interpret displacement–time graphs and velocity–time graphs, and in particular appreciate that
 - the area under a velocity–time graph represents displacement,
 - the gradient of a displacement–time graph represents velocity,
 - the gradient of a velocity–time graph represents acceleration
- use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration
- use appropriate formulae for motion with constant acceleration in a straight line.

Notes and examples

Restricted to motion in one dimension only.
The term 'deceleration' may sometimes be used in the context of decreasing speed.

Calculus required is restricted to techniques from the content for Paper 1: Pure Mathematics 1.

Questions may involve setting up more than one equation, using information about the motion of different particles.

4.3 Momentum

Candidates should be able to:

- use the definition of linear momentum and show understanding of its vector nature
- use conservation of linear momentum to solve problems that may be modelled as the direct impact of two bodies.

Notes and examples

For motion in one dimension only.

Including direct impact of two bodies where the bodies coalesce on impact.

Knowledge of impulse and the coefficient of restitution is not required.

4 Mechanics

4.4 Newton's laws of motion

Candidates should be able to:

- apply Newton's laws of motion to the linear motion of a particle of constant mass moving under the action of constant forces, which may include friction, tension in an inextensible string and thrust in a connecting rod
- use the relationship between mass and weight
- solve simple problems which may be modelled as the motion of a particle moving vertically or on an inclined plane with constant acceleration
- solve simple problems which may be modelled as the motion of connected particles.

Notes and examples

If any other forces resisting motion are to be considered (e.g. air resistance) this will be indicated in the question.

$W = mg$. In this component, questions are mainly numerical, and use of the approximate numerical value $10 \text{ (ms}^{-2}\text{)}$ for g is expected.

Including, for example, motion of a particle on a rough plane where the acceleration while moving up the plane is different from the acceleration while moving down the plane.

e.g. particles connected by a light inextensible string passing over a smooth pulley, or a car towing a trailer by means of either a light rope or a light rigid tow-bar.

4.5 Energy, work and power

Candidates should be able to:

- understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force
- understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae
- understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy
- use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion
- solve problems involving, for example, the instantaneous acceleration of a car moving on a hill against a resistance.

Notes and examples

$$W = Fd \cos \theta;$$

Use of the scalar product is not required.

Including cases where the motion may not be linear (e.g. a child on a smooth curved 'slide'), where only overall energy changes need to be considered.

Including calculation of (average) power as

$$\frac{\text{Work done}}{\text{Time taken}}$$

$$P = Fv.$$

5 Probability & Statistics 1 (for Paper 5)

Questions set will be mainly numerical, and will test principles in probability and statistics without involving knowledge of algebraic methods beyond the content for Paper 1: Pure Mathematics 1.

Knowledge of the following probability notation is also assumed: $P(A)$, $P(A \cup B)$, $P(A \cap B)$, $P(A|B)$ and the use of A' to denote the complement of A .

5.1 Representation of data

Candidates should be able to:

- select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have
- draw and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency graphs
- understand and use different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation)
- use a cumulative frequency graph
- calculate and use the mean and standard deviation of a set of data (including grouped data) either from the data itself or from given totals $\sum x$ and $\sum x^2$, or coded totals $\sum(x - a)$ and $\sum(x - a)^2$, and use such totals in solving problems which may involve up to two data sets.

Notes and examples

Including back-to-back stem-and-leaf diagrams.

e.g. in comparing and contrasting sets of data.

e.g. to estimate medians, quartiles, percentiles, the proportion of a distribution above (or below) a given value, or between two values.

5.2 Permutations and combinations

Candidates should be able to:

- understand the terms permutation and combination, and solve simple problems involving selections
- solve problems about arrangements of objects in a line, including those involving
 - repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS')
 - restriction (e.g. the number of ways several people can stand in a line if two particular people must, or must not, stand next to each other).

Notes and examples

Questions may include cases such as people sitting in two (or more) rows.

Questions about objects arranged in a circle will not be included.

5 Probability & Statistics 1

5.3 Probability

Candidates should be able to:

- evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events, or by calculation using permutations or combinations
- use addition and multiplication of probabilities, as appropriate, in simple cases
- understand the meaning of exclusive and independent events, including determination of whether events A and B are independent by comparing the values of $P(A \cap B)$ and $P(A) \times P(B)$
- calculate and use conditional probabilities in simple cases.

Notes and examples

e.g. the total score when two fair dice are thrown.
e.g. drawing balls at random from a bag containing balls of different colours.

Explicit use of the general formula

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ is not required.

e.g. situations that can be represented by a sample space of equiprobable elementary events, or a tree diagram. The use of $P(A|B) = \frac{P(A \cap B)}{P(B)}$ may be required in simple cases.

5.4 Discrete random variables

Candidates should be able to:

- draw up a probability distribution table relating to a given situation involving a discrete random variable X , and calculate $E(X)$ and $\text{Var}(X)$
- use formulae for probabilities for the binomial and geometric distributions, and recognise practical situations where these distributions are suitable models
- use formulae for the expectation and variance of the binomial distribution and for the expectation of the geometric distribution.

Notes and examples

Including the notations $B(n, p)$ and $\text{Geo}(p)$. $\text{Geo}(p)$ denotes the distribution in which $p_r = p(1-p)^{r-1}$ for $r = 1, 2, 3, \dots$.

Proofs of formulae are not required.

5 Probability & Statistics 1

5.5 The normal distribution

Candidates should be able to:

- understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables
- solve problems concerning a variable X , where $X \sim N(\mu, \sigma^2)$, including
 - finding the value of $P(X > x_1)$, or a related probability, given the values of x_1 , μ , σ .
 - finding a relationship between x_1 , μ and σ given the value of $P(X > x_1)$ or a related probability
- recall conditions under which the normal distribution can be used as an approximation to the binomial distribution, and use this approximation, with a continuity correction, in solving problems.

Notes and examples

Sketches of normal curves to illustrate distributions or probabilities may be required.

For calculations involving standardisation, full details of the working should be shown.

$$\text{e.g. } Z = \frac{(X - \mu)}{\sigma}$$

n sufficiently large to ensure that both $np > 5$ and $nq > 5$.

6 Probability & Statistics 2 (for Paper 6)

Knowledge of the content of Paper 5: Probability & Statistics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions. Knowledge of calculus within the content for Paper 3: Pure Mathematics 3 will also be assumed.

6.1 The Poisson distribution

Candidates should be able to:

- use formulae to calculate probabilities for the distribution $Po(\lambda)$
- use the fact that if $X \sim Po(\lambda)$ then the mean and variance of X are each equal to λ
- understand the relevance of the Poisson distribution to the distribution of random events, and use the Poisson distribution as a model
- use the Poisson distribution as an approximation to the binomial distribution where appropriate
- use the normal distribution, with continuity correction, as an approximation to the Poisson distribution where appropriate.

Notes and examples

Proofs are not required.

The conditions that n is large and p is small should be known; $n > 50$ and $np < 5$, approximately.

The condition that λ is large should be known; $\lambda > 15$, approximately.

6.2 Linear combinations of random variables

Candidates should be able to:

- use, when solving problems, the results that
 - $E(aX + b) = aE(X) + b$ and $\text{Var}(aX + b) = a^2 \text{Var}(X)$
 - $E(aX + bY) = aE(X) + bE(Y)$
 - $\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$ for independent X and Y
 - if X has a normal distribution then so does $aX + b$
 - if X and Y have independent normal distributions then $aX + bY$ has a normal distribution
 - if X and Y have independent Poisson distributions then $X + Y$ has a Poisson distribution.

Notes and examples

Proofs of these results are not required.

6 Probability & Statistics 2

6.3 Continuous random variables

Candidates should be able to:

- understand the concept of a continuous random variable, and recall and use properties of a probability density function
- use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a distribution.

Notes and examples

For density functions defined over a single interval only; the domain may be infinite,

e.g. $\frac{3}{x^4}$ for $x \geq 1$.

Including location of the median or other percentiles of a distribution by direct consideration of an area using the density function.

Explicit knowledge of the cumulative distribution function is not included.

6.4 Sampling and estimation

Candidates should be able to:

- understand the distinction between a sample and a population, and appreciate the necessity for randomness in choosing samples
- explain in simple terms why a given sampling method may be unsatisfactory
- recognise that a sample mean can be regarded as a random variable, and use the facts that $E(\bar{X}) = \mu$ and that $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$
- use the fact that (\bar{X}) has a normal distribution if X has a normal distribution
- use the Central Limit Theorem where appropriate
- calculate unbiased estimates of the population mean and variance from a sample, using either raw or summarised data
- determine and interpret a confidence interval for a population mean in cases where the population is normally distributed with known variance or where a large sample is used
- determine, from a large sample, an approximate confidence interval for a population proportion.

Notes and examples

Including an elementary understanding of the use of random numbers in producing random samples.

Knowledge of particular sampling methods, such as quota or stratified sampling, is not required.

Only an informal understanding of the Central Limit Theorem (CLT) is required; for large sample sizes, the distribution of a sample mean is approximately normal.

Only a simple understanding of the term 'unbiased' is required, e.g. that although individual estimates will vary the process gives an accurate result 'on average'.

6 Probability & Statistics 2

6.5 Hypothesis tests

Candidates should be able to:

- understand the nature of a hypothesis test, the difference between one-tailed and two-tailed tests, and the terms null hypothesis, alternative hypothesis, significance level, rejection region (or critical region), acceptance region and test statistic
- formulate hypotheses and carry out a hypothesis test in the context of a single observation from a population which has a binomial or Poisson distribution, using
 - direct evaluation of probabilities
 - a normal approximation to the binomial or the Poisson distribution, where appropriate
- formulate hypotheses and carry out a hypothesis test concerning the population mean in cases where the population is normally distributed with known variance or where a large sample is used
- understand the terms Type I error and Type II error in relation to hypothesis tests
- calculate the probabilities of making Type I and Type II errors in specific situations involving tests based on a normal distribution or direct evaluation of binomial or Poisson probabilities.

Notes and examples

Outcomes of hypothesis tests are expected to be interpreted in terms of the contexts in which questions are set.

4 Details of the assessment

Relationship between components

Candidates build their knowledge of the mathematics content as they progress through the course.

Paper 1: Pure Mathematics 1 is the foundation for all other components.

Paper 2: Pure Mathematics 2 and Paper 3: Pure Mathematics 3 build on the subject content for Paper 1: Pure Mathematics 1.

Paper 4: Mechanics and Paper 5: Probability & Statistics 1 components assume prior knowledge of the Paper 1: Pure Mathematics 1 content.

Paper 5: Probability & Statistics 1 is the foundation for studying Paper 6: Probability & Statistics 2.

Candidates may not take both Paper 2 and Paper 3 in the same examination series. Paper 2 and Paper 3 are taken in alternative routes through the qualification – Paper 2 is for AS Level only, and Paper 3 is for A Level. Paper 2 subject content is largely a subset of the Paper 3 subject content.

Examination information

All components are assessed by written examinations which are externally marked. Sample assessment materials are available on our website at www.cambridgeinternational.org showing the question style and level of the examination papers.

Application of mathematical techniques

As well as demonstrating the appropriate techniques, candidates need to apply their knowledge in solving problems. Individual examination questions may involve ideas and methods from more than one section of the subject content for that component.

The main focus of examination questions will be the AS & A Level Mathematics subject content. However, in examination questions, candidates may need to make use of prior knowledge and mathematical techniques from previous study, as listed in section 3 of this syllabus.

Structure of the question paper

All questions in the examination papers are compulsory. An approximate number of questions for each paper is given in the Assessment overview in section 2 of this syllabus. Questions are of varied lengths and often contain several parts, labelled (a), (b), (c), which may have sub-parts (i), (ii), (iii), as needed. Some questions might require candidates to sketch graphs or diagrams, or draw accurate graphs.

Answer space

Candidates answer on the question paper. All working should be shown neatly and clearly in the spaces provided for each question. New questions often start on a fresh page, so more answer space may be provided than is needed. If additional space is required, candidates should use the lined page at the end of the question paper, where the question number or numbers must be clearly shown.

Degrees of accuracy

Candidates should give non-exact numerical answers correct to three significant figures (or one decimal place for angles in degrees) unless a different level of accuracy is specified in the question. To earn accuracy marks, candidates should avoid rounding figures until they have their final answer.

Additional materials for examinations

Candidates are expected to have the following equipment in examinations:

- a ruler
- a scientific calculator (see the following section).

Note: a protractor and a pair of compasses are **not** required.

A list of formulae and statistical tables (MF19) is supplied in examinations for the use of candidates. A copy of the list of formulae and tables is given for reference in section 5 of this syllabus. Note that MF19 is a combined formulae list for AS & A Level Mathematics (9709) and AS & A Level Further Mathematics (9231). Some formulae in the list are not needed for this syllabus, and are only for Further Mathematics (9231); these are listed in separate sections labelled Further Pure Mathematics, Further Mechanics, and Further Probability & Statistics.

Calculators

It is expected that candidates will have a calculator with standard 'scientific' functions available for use in all the examinations. Computers, graphical calculators and calculators capable of symbolic algebraic manipulation or symbolic differentiation or integration are not permitted. The General Regulations concerning the use of calculators are contained in the *Cambridge Handbook* at www.cambridgeinternational.org/examsOfficers

Candidates are expected to show all necessary working; no marks will be given for unsupported answers from a calculator.

Mathematical notation

The list of mathematical notation that may be used in examinations for this syllabus is available on our website at www.cambridgeinternational.org/9709

Command words

The table below includes command words used in the assessment for this syllabus. The use of the command word will relate to the subject context.

Command word	What it means
Calculate	work out from given facts, figures or information
Describe	state the points of a topic / give characteristics and main features
Determine	establish with certainty
Evaluate	judge or calculate the quality, importance, amount, or value of something
Explain	set out purposes or reasons / make the relationships between things evident / provide why and/or how and support with relevant evidence
Identify	name/select/recognise
Justify	support a case with evidence/argument
Show (that)	provide structured evidence that leads to a given result
Sketch	make a simple freehand drawing showing the key features
State	express in clear terms
Verify	confirm a given statement/result is true

5 List of formulae and statistical tables (MF19)

PURE MATHEMATICS

Mensuration

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Volume of cone or pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$$

$$\text{Area of curved surface of cone} = \pi r \times \text{slant height}$$

$$\text{Arc length of circle} = r\theta \quad (\theta \text{ in radians})$$

$$\text{Area of sector of circle} = \frac{1}{2}r^2\theta \quad (\theta \text{ in radians})$$

Algebra

For the quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For an arithmetic series:

$$u_n = a + (n-1)d, \quad S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

For a geometric series:

$$u_n = ar^{n-1}, \quad S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1), \quad S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

Binomial series:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + b^n, \text{ where } n \text{ is a positive integer}$$

$$\text{and } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots, \text{ where } n \text{ is rational and } |x| < 1$$

Trigonometry

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$$

$$\cos^2 \theta + \sin^2 \theta \equiv 1, \quad 1 + \tan^2 \theta \equiv \sec^2 \theta, \quad \cot^2 \theta + 1 \equiv \operatorname{cosec}^2 \theta$$

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2 \cos^2 A - 1 \equiv 1 - 2 \sin^2 A$$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$

Principal values:

$$-\frac{1}{2}\pi \leq \sin^{-1} x \leq \frac{1}{2}\pi, \quad 0 \leq \cos^{-1} x \leq \pi, \quad -\frac{1}{2}\pi < \tan^{-1} x < \frac{1}{2}\pi$$

Differentiation

f(x)	f'(x)
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
uv	$v \frac{du}{dx} + u \frac{dv}{dx}$
$\frac{u}{v}$	$\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

$$\text{If } x = f(t) \text{ and } y = g(t) \text{ then } \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

Integration(Arbitrary constants are omitted; a denotes a positive constant.)

$f(x)$	$\int f(x) dx$	
x^n	$\frac{x^{n+1}}{n+1}$	$(n \neq -1)$
$\frac{1}{x}$	$\ln x $	
e^x	e^x	
$\sin x$	$-\cos x$	
$\cos x$	$\sin x$	
$\sec^2 x$	$\tan x$	
$\frac{1}{x^2 + a^2}$	$\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$	
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a} \ln\left \frac{x-a}{x+a}\right $	$(x > a)$
$\frac{1}{a^2 - x^2}$	$\frac{1}{2a} \ln\left \frac{a+x}{a-x}\right $	$(x < a)$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)|$$

*Vectors*If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}| |\mathbf{b}| \cos \theta$$

FURTHER PURE MATHEMATICS

Algebra

Summations:

$$\sum_{r=1}^n r = \frac{1}{2}n(n+1), \quad \sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1), \quad \sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

Maclaurin's series:

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \dots + \frac{x^r}{r!}f^{(r)}(0) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad (\text{all } x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad (\text{all } x)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad (\text{all } x)$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 \leq x \leq 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2r+1}}{(2r+1)!} + \dots \quad (\text{all } x)$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2r}}{(2r)!} + \dots \quad (\text{all } x)$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 < x < 1)$$

Trigonometry

If $t = \tan \frac{1}{2}x$ then:

$$\sin x = \frac{2t}{1+t^2} \quad \text{and} \quad \cos x = \frac{1-t^2}{1+t^2}$$

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x \equiv 1, \quad \sinh 2x \equiv 2 \sinh x \cosh x, \quad \cosh 2x \equiv \cosh^2 x + \sinh^2 x$$

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}) \quad (x \geq 1)$$

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (|x| < 1)$$

Differentiation

$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$

Integration(Arbitrary constants are omitted; a denotes a positive constant.)

$f(x)$	$\int f(x) dx$
$\sec x$	$\ln \sec x + \tan x = \ln \tan(\frac{1}{2}x + \frac{1}{4}\pi) $ ($ x < \frac{1}{2}\pi$)
$\operatorname{cosec} x$	$-\ln \operatorname{cosec} x + \cot x = \ln \tan(\frac{1}{2}x) $ ($0 < x < \pi$)
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\operatorname{sech}^2 x$	$\tanh x$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$ ($ x < a$)
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right)$ ($x > a$)
$\frac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}\left(\frac{x}{a}\right)$

MECHANICS*Uniformly accelerated motion*

$$v = u + at, \quad s = \frac{1}{2}(u + v)t, \quad s = ut + \frac{1}{2}at^2, \quad v^2 = u^2 + 2as$$

FURTHER MECHANICS*Motion of a projectile*

Equation of trajectory is:

$$y = x \tan \theta - \frac{gx^2}{2V^2 \cos^2 \theta}$$

Elastic strings and springs

$$T = \frac{\lambda x}{l}, \quad E = \frac{\lambda x^2}{2l}$$

Motion in a circle

For uniform circular motion, the acceleration is directed towards the centre and has magnitude

$$\omega^2 r \quad \text{or} \quad \frac{v^2}{r}$$

*Centres of mass of uniform bodies*Triangular lamina: $\frac{2}{3}$ along median from vertexSolid hemisphere of radius r : $\frac{3}{8}r$ from centreHemispherical shell of radius r : $\frac{1}{2}r$ from centreCircular arc of radius r and angle 2α : $\frac{r \sin \alpha}{\alpha}$ from centreCircular sector of radius r and angle 2α : $\frac{2r \sin \alpha}{3\alpha}$ from centreSolid cone or pyramid of height h : $\frac{3}{4}h$ from vertex

PROBABILITY & STATISTICS*Summary statistics*

For ungrouped data:

$$\bar{x} = \frac{\Sigma x}{n}, \quad \text{standard deviation} = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

For grouped data:

$$\bar{x} = \frac{\Sigma xf}{\Sigma f}, \quad \text{standard deviation} = \sqrt{\frac{\Sigma(x - \bar{x})^2 f}{\Sigma f}} = \sqrt{\frac{\Sigma x^2 f}{\Sigma f} - \bar{x}^2}$$

Discrete random variables

$$E(X) = \Sigma xp, \quad \text{Var}(X) = \Sigma x^2 p - \{E(X)\}^2$$

For the binomial distribution $B(n, p)$:

$$p_r = \binom{n}{r} p^r (1-p)^{n-r}, \quad \mu = np, \quad \sigma^2 = np(1-p)$$

For the geometric distribution $\text{Geo}(p)$:

$$p_r = p(1-p)^{r-1}, \quad \mu = \frac{1}{p}$$

For the Poisson distribution $\text{Po}(\lambda)$

$$p_r = e^{-\lambda} \frac{\lambda^r}{r!}, \quad \mu = \lambda, \quad \sigma^2 = \lambda$$

Continuous random variables

$$E(X) = \int x f(x) dx, \quad \text{Var}(X) = \int x^2 f(x) dx - \{E(X)\}^2$$

Sampling and testing

Unbiased estimators:

$$\bar{x} = \frac{\Sigma x}{n}, \quad s^2 = \frac{\Sigma(x - \bar{x})^2}{n-1} = \frac{1}{n-1} \left(\Sigma x^2 - \frac{(\Sigma x)^2}{n} \right)$$

Central Limit Theorem:

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

Approximate distribution of sample proportion:

$$N\left(p, \frac{p(1-p)}{n}\right)$$

FURTHER PROBABILITY & STATISTICS*Sampling and testing*

Two-sample estimate of a common variance:

$$s^2 = \frac{\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

Probability generating functions

$$G_X(t) = E(t^X),$$

$$E(X) = G'_X(1),$$

$$\text{Var}(X) = G''_X(1) + G'_X(1) - \{G'_X(1)\}^2$$

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1, then, for each value of z , the table gives the value of $\Phi(z)$, where

$$\Phi(z) = P(Z \leq z).$$



For negative values of z , use $\Phi(-z) = 1 - \Phi(z)$.

z											ADD								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1, then, for each value of p , the table gives the value of z such that

$$P(Z \leq z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE t -DISTRIBUTION

If T has a t -distribution with ν degrees of freedom, then, for each pair of values of p and ν , the table gives the value of t such that:

$$P(T \leq t) = p.$$



p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$\nu=1$	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE χ^2 -DISTRIBUTION

If X has a χ^2 -distribution with ν degrees of freedom then, for each pair of values of p and ν , the table gives the value of x such that

$$P(X \leq x) = p.$$



p	0.01	0.025	0.05	0.9	0.95	0.975	0.99	0.995	0.999
$\nu=1$	0.0 ³ 1571	0.0 ³ 9821	0.0 ² 3932	2.706	3.841	5.024	6.635	7.879	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.51
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.647	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

WILCOXON SIGNED-RANK TEST

The sample has size n .

P is the sum of the ranks corresponding to the positive differences.

Q is the sum of the ranks corresponding to the negative differences.

T is the smaller of P and Q .

For each value of n the table gives the **largest** value of T which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of T

	Level of significance			
	0.05	0.025	0.01	0.005
One-tailed	0.05	0.025	0.01	0.005
Two-tailed	0.1	0.05	0.02	0.01
$n = 6$	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37

For larger values of n , each of P and Q can be approximated by the normal distribution with mean $\frac{1}{4}n(n+1)$ and variance $\frac{1}{24}n(n+1)(2n+1)$.

WILCOXON RANK-SUM TEST

The two samples have sizes m and n , where $m \leq n$.

R_m is the sum of the ranks of the items in the sample of size m .

W is the smaller of R_m and $m(n + m + 1) - R_m$.

For each pair of values of m and n , the table gives the **largest** value of W which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of W

	Level of significance											
One-tailed	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two-tailed	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 3$			$m = 4$			$m = 5$			$m = 6$		
3	6	–	–									
4	6	–	–	11	10	–						
5	7	6	–	12	11	10	19	17	16			
6	8	7	–	13	12	11	20	18	17	28	26	24
7	8	7	6	14	13	11	21	20	18	29	27	25
8	9	8	6	15	14	12	23	21	19	31	29	27
9	10	8	7	16	14	13	24	22	20	33	31	28
10	10	9	7	17	15	13	26	23	21	35	32	29

	Level of significance											
One-tailed	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two-tailed	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 7$			$m = 8$			$m = 9$			$m = 10$		
7	39	36	34									
8	41	38	35	51	49	45						
9	43	40	37	54	51	47	66	62	59			
10	45	42	39	56	53	49	69	65	61	82	78	74

For larger values of m and n , the normal distribution with mean $\frac{1}{2}m(m + n + 1)$ and variance $\frac{1}{12}mn(m + n + 1)$ should be used as an approximation to the distribution of R_m .

6 What else you need to know

This section is an overview of other information you need to know about this syllabus. It will help to share the administrative information with your exams officer so they know when you will need their support. Find more information about our administrative processes at www.cambridgeinternational.org/examsofficers

Before you start

Previous study

We recommend that learners starting this course should have studied a Cambridge IGCSE Mathematics (Extended) or Cambridge O Level course or the equivalent. See the introduction to section 3 of this syllabus for more details of expected prior knowledge.

Guided learning hours

We design Cambridge International AS & A Level syllabuses based on learners having about 180 guided learning hours for each Cambridge International AS Level and about 360 guided learning hours for a Cambridge International A Level. The number of hours a learner needs to achieve the qualification may vary according to local practice and their previous experience of the subject.

Availability and timetables

All Cambridge schools are allocated to one of six administrative zones. Each zone has a specific timetable.

You can view the timetable for your administrative zone at www.cambridgeinternational.org/timetables

You can enter candidates in the June and November exam series. If your school is in India, you can enter your candidates in the March exam series.

Check you are using the syllabus for the year the candidate is taking the exam.

Private candidates can enter for this syllabus.

Combining with other syllabuses

Candidates can take this syllabus alongside other Cambridge International syllabuses in a single exam series. The only exceptions are:

- syllabuses with the same title at the same level.

Note that candidates can take AS & A Level Mathematics (9709) in the same exam series with AS & A Level Further Mathematics (9231).

Group awards: Cambridge AICE

Cambridge AICE (Advanced International Certificate of Education) is a group award for Cambridge International AS & A Level. It allows schools to offer a broad and balanced curriculum by recognising the achievements of learners who pass examinations in a range of different subjects.

Learn more about Cambridge AICE at www.cambridgeinternational.org/aice

Making entries

Exams officers are responsible for submitting entries to Cambridge International. We encourage them to work closely with you to make sure they enter the right number of candidates for the right combination of syllabus components. Entry option codes and instructions for submitting entries are in the *Cambridge Guide to Making Entries*. Your exams officer has a copy of this guide.

Exam administration

To keep our exams secure, we produce question papers for different areas of the world, known as administrative zones. We allocate all Cambridge schools to one administrative zone determined by their location. Each zone has a specific timetable. Some of our syllabuses offer candidates different assessment options. An entry option code is used to identify the components the candidate will take relevant to the administrative zone and the available assessment options.

Support for exams officers

We know how important exams officers are to the successful running of exams. We provide them with the support they need to make your entries on time. Your exams officer will find this support, and guidance for all other phases of the Cambridge Exams Cycle, at www.cambridgeinternational.org/eoguide

Retakes

Candidates can retake Cambridge International AS Level and Cambridge International A Level as many times as they want to. Information on retake entries is at www.cambridgeinternational.org/entries. To confirm what entry options are available for this syllabus, refer to the *Cambridge Guide to Making Entries* for the relevant series.

Equality and inclusion

We have taken great care to avoid bias of any kind in the preparation of this syllabus and related assessment materials. In our effort to comply with the UK Equality Act (2010) we have taken all reasonable steps to avoid any direct and indirect discrimination.

The standard assessment arrangements may present barriers for candidates with impairments. Where a candidate is eligible, we may be able to make arrangements to enable that candidate to access assessments and receive recognition of their attainment. We do not agree access arrangements if they give candidates an unfair advantage over others or if they compromise the standards being assessed.

Information on access arrangements is in the *Cambridge Handbook* at www.cambridgeinternational.org/eoguide

Language

This syllabus and the related assessment materials are available in English only.

After the exam

Grading and reporting

Grades A*, A, B, C, D or E indicate the standard a candidate achieved at Cambridge International A Level. A* is the highest and E is the lowest grade.

Grades a, b, c, d or e indicate the standard a candidate achieved at Cambridge International AS Level. 'a' is the highest and 'e' is the lowest grade.

'Ungraded' means that the candidate's performance did not meet the standard required for the lowest grade (E or e). 'Ungraded' is reported on the statement of results but not on the certificate. In specific circumstances your candidates may see one of the following letters on their statement of results:

- Q (PENDING)
- X (NO RESULT).

These letters do not appear on the certificate.

If a candidate takes a Cambridge International A Level and fails to achieve grade E or higher, a Cambridge International AS Level grade will be awarded if both of the following apply:

- the components taken for the Cambridge International A Level by the candidate in that series included all the components making up a Cambridge International AS Level
- the candidate's performance on the AS Level components was sufficient to merit the award of a Cambridge International AS Level grade.

On the statement of results and certificates, Cambridge International AS & A Levels are shown as General Certificates of Education, GCE Advanced Subsidiary Level (GCE AS Level) and GCE Advanced Level (GCE A Level).

'Cambridge International A Levels are the 'gold standard' qualification. They are based on rigorous, academic syllabuses that are accessible to students from a wide range of abilities yet have the capacity to stretch our most able.'

Mark Vella, Director of Studies, Auckland Grammar School, New Zealand

How students, teachers and higher education can use the grades

Cambridge International A Level

Assessment at Cambridge International A Level has two purposes:

- to measure learning and achievement

The assessment:

- confirms achievement and performance in relation to the knowledge, understanding and skills specified in the syllabus, to the levels described in the grade descriptions.

- to show likely future success

The outcomes:

- help predict which students are well prepared for a particular course or career and/or which students are more likely to be successful
- help students choose the most suitable course or career.

Cambridge International AS Level

Assessment at Cambridge International AS Level has two purposes:

- to measure learning and achievement

The assessment:

- confirms achievement and performance in relation to the knowledge, understanding and skills specified in the syllabus.

- to show likely future success

The outcomes:

- help predict which students are well prepared for a particular course or career and/or which students are more likely to be successful
- help students choose the most suitable course or career
- help decide whether students part way through a Cambridge International A Level course are making enough progress to continue
- guide teaching and learning in the next stages of the Cambridge International A Level course.

Grade descriptions

Grade descriptions are provided to give an indication of the standards of achievement candidates awarded particular grades are likely to show. Weakness in one aspect of the examination may be balanced by a better performance in some other aspect.

Grade descriptions for Cambridge International AS & Level Mathematics will be published after the first assessment of the A Level in 2020. Find more information at www.cambridgeinternational.org/alevel

Changes to this syllabus for 2023, 2024 and 2025

The syllabus has been updated. This is version 1, published September 2020.

You must read the whole syllabus before planning your teaching programme.

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- Changes to syllabus content**
- Information on routes through the qualification on pages 8–13 has been updated to improve clarity for schools.
-

Significant changes to the syllabus are indicated by black vertical lines either side of the text.

Any textbooks endorsed to support the syllabus for examination from 2020 are still suitable for use with this syllabus.



'While studying Cambridge IGCSE and Cambridge International A Levels, students broaden their horizons through a global perspective and develop a lasting passion for learning.'

Zhai Xiaoning, Deputy Principal, The High School Affiliated to Renmin University of China

Cambridge Assessment International Education
The Triangle Building, Shaftesbury Road, Cambridge, CB2 8EA, United Kingdom
Tel: +44 (0)1223 553554 Fax: +44 (0)1223 553558
Email: info@cambridgeinternational.org www.cambridgeinternational.org

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